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DIRECTIONAL ANTENNAS
IN THE
STANDARD BROADCAST BAND

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Introduction

Engineers in the Standard Broadcast Division of the Commission are frequently confronted with the problem of checking the proposed design of a directional antenna array for use in the AM broadcast band. Many articles have been written on the subject covering the design theory. It is proposed in this paper to collect and present in useful form the formulas, concepts, and available curves which are needed to check the design of an array.

The usual assumptions of perfect ground and sinusoidal tower current distribution are made. For the performance of a directional array the field intensity at one mile in the various directions is studied. The fields at other distances are easily obtained from the appropriate conductivity curves in the Standards.

Basic Formulas Relating to Array Power, RMS, and Design Constants

In this section basic formulas will be developed for checking the RMS field and the pattern constants of an array.

The RMS field for any angle of elevation, θ, is the equivalent non-directional field intensity which would have the same power radiated between the cones at θ and $\theta + d\theta$. The RMS field for any angle of elevation, θ, is given by:

$$E^2_{RMS}(\theta) = \sum_{n=1}^{N} \sum_{p=1}^{N} E_n E_p \, f_n(\theta) \, f_p(\theta) \, J_o(S_{np} \cos \theta) \cos (\psi_n - \psi_p)$$

(1)

$$= E_o^2 \sum_{n=1}^{N} \sum_{p=1}^{N} F_n F_p \, f_n(\theta) \, f_p(\theta) \, J_o(S_{np} \cos \theta) \cos (\psi_n - \psi_p)$$

where,

E_o = horizontal field intensity at 1 mile from the reference tower in mv/m.

E_n = horizontal field intensity at 1 mile from tower n in mv/m.

F_n = the horizontal field ratio E_n/E_o.

ψ_n = the phase angle of current in tower n, referred to the current in the reference tower.

S_{np} = the spacing between towers n and p in electrical degrees.

$J_o(S_{np}\cos\theta)$ = Bessel function of $S_{np} \cos \theta$.

$f_n(\theta)$ = antenna form factor defined by equation 19 and plotted in Fig. 3.

However, the RMS field is seldom required for any angle of elevation except the horizontal which contains the useful radiated power. The horizontal RMS is,

$$E^2_{RMS} = \sum_{n=1}^{N} \sum_{p=1}^{N} E_n E_p \, J_o(S_{np}) \cos(\psi_n - \psi_p)$$

$$= E_o^2 \sum_{n=1}^{N} \sum_{p=1}^{N} F_n F_p \, J_o(S_{np}) \cos(\psi_n - \psi_p)$$

(1a)

$$= E_1{}^2 + E_2{}^2 + \ldots\ldots\ldots\ldots + E_N{}^2$$

$$+ 2 E_1 E_2 J_o(S_{12}) \cos(\psi_1 - \psi_2) + \ldots 2 E_1 E_N J_o(S_{1N}) \cos(\psi_1 - \psi_N)$$

$$+ 2 E_2 E_3 J_o(S_{23}) \cos(\psi_2 - \psi_3) + \ldots 2 E_2 E_N J_o(S_{2N}) \cos(\psi_2 - \psi_N)$$

$$+ \ldots\ldots\ldots$$

$$+ 2 E_{N-1} E_N J_o \left[S_{(N-1)N}\right] \cos(\psi_{N-1} - \psi_N)$$

The horizontal field intensity E_n from any tower is related to the loop current I_n in that tower by,

(2) $$E_n = 37.25 \, I_n \, (1 - \cos G_n)$$

where G_n is the antenna height in electrical degrees of the nth tower.

Another basic formula is the total power radiated by the system in all directions:

$$P_R = \sum_{n=1}^{N} \sum_{p=1}^{N} I_n I_p R_{np} \cos(\psi_n - \psi_p)$$

(3)

$$= I_o{}^2 R_{oo} \frac{\sum_{n=1}^{N} \sum_{p=1}^{N} M_n M_p R_{np} \cos(\psi_n - \psi_p)}{R_{oo}}$$

$$= I_1^2 R_{11} + I_2^2 R_{22} + \ldots\ldots I_N^2 R_{NN}$$

$$+ 2I_1 I_2 R_{12} \cos(\psi_1 - \psi_2) + \ldots\ldots + 2I_1 I_N R_{1N} \cos(\psi_1 - \psi_N)$$

$$+ 2I_2 I_3 R_{23} \cos(\psi_2 - \psi_3) + \ldots\ldots + 2I_2 I_N R_{2N} \cos(\psi_2 - \psi_N)$$

$$+ \ldots\ldots\ldots$$

$$+ 2I_{N-1} I_N R_{(N-1)N} \cos(\psi_{N-1} - \psi_N)$$

In the above expression,

I_o = loop current in reference tower.

I_n = loop current in tower n.

M_n = the current ratio I_n/I_o.

R_{oo} = self resistance of reference tower.

R_{nn} = self resistance of tower n.

R_{np} = mutual resistance between towers n and p.

It is noted that the reference tower may be imaginary to realize the simplest numbers for the given ratios, phases, spaces, etc., but in order for the results to have significance the reference height G_o should be equal to one of the G_n or possibly an average or mean of all the heights.

The power loss due to an imperfect ground system and the dissipation loss in the towers themselves is given by,

$$P_L = \sum_{n=1}^{N} I_n^2 R_{Ln} \sin G_n^2$$

(4)

$$= I_1^2 R_{L1} \sin^2 G_1 + I_2^2 R_{L2} \sin^2 G_2 + \ldots + I_N^2 R_{LN} \sin^2 G_N$$

where R_{Ln} is the base tower loss resistance which combines the effective ground resistance and ohmic loss in the nth tower. R_{LN} is usually assumed the same for all towers at some value between 0.5 and 4 ohms, usually about 2 ohms. A note of caution should be added here that this assumption is not necessarily accurate and that differences in R_{Ln} at the various towers might conceivably be important in exceptional cases where the towers carry greatly different currents. However, for checking a design, the assumption of a constant R_{Ln} is nearly always good enough, especially since there is not usually sufficient data to make a more accurate determination of the individual R_{Ln}. For the unusual case where different R_{Ln} values are considered, adjustments can be made in the derived formulas. With the assumption of constant R_{Ln}, equation 4 reduces to,

$$P_L = I_o^2 R_{oo} \left[\frac{R_L}{R_{oo}} \sum_{n=1}^{N} M_n^2 \sin^2 G_n \right] \tag{4a}$$

It is noted that for antennas approximately one half wavelength in height the loop resistance value of the tower and ground loss; i.e., $R_L \sin^2 G_n$—approaches zero, giving a physically incorrect answer. For this case some value in place of $\sin G_n$ should be assigned to give a reasonable loop loss resistance in equations (4) and (4a). For these high antennas the loss resistance is small compared to the self resistance of the tower so that usually the effect of R_L is of minor importance.

The total power into the towers of the array is:

$$P = P_R + P_L \tag{5}$$

From equation (2) the loop current and field ratios are related by:

$$F_n = M_n \left[\frac{1 - \cos G_n}{1 - \cos G_o} \right] \tag{6}$$

Introducing the following notation:

$$\left\{ \begin{aligned} e^2 &= \sum_{n=1}^{N} \sum_{p=1}^{N} F_n F_p J_o(S_{np}) \cos (\psi_n - \psi_p) \\ r^2 &= \frac{\sum_{n=1}^{N} \sum_{p=1}^{N} M_n M_p R_{np} \cos (\psi_n - \psi_p)}{R_{oo}} \\ a^2 &= \frac{\sum_{n=1}^{N} M_n^2 \sin^2 G_n}{R_{oo}} \\ r_L^2 &= r^2 + a^2 R_L \end{aligned} \right. \tag{7}$$

equations (1a), (2), (3), (4a) and (5) become:

(1b) $E_{RMS} = E_o\ e$

(2a) $E_o = 37.25\ I_o(1 - \cos G_o)$

(3a) $P_R = I_o^2\ R_{oo}\ r^2$

(4b) $P_L = I_o^2\ R_{oo}\ a^2\ R_L$

(5a) $P = I_o^2\ R_{oo}\ (r^2 + a^2\ R_L)$

$\quad = I_o^2\ R_{oo}\ r_L^2$

The above list of basic formulas together with the definitions for e, r and a constitutes a simple but complete set of equations for the solution and discussion of the array power, the RMS, all the current and field magnitudes, and the power efficiency. For example, given the array design the following steps could be taken for the solution:

a. Compute e, a, r, and r_L from (7).

b. Estimate a proper value for R_L and then with (5a) compute the magnitude of the reference current I_o and consequently the magnitude of every other current.

c. With (2a) compute the reference field E_o and so draw up the pattern in mv/m.

d. The expected RMS may be calculated from (1b).

It is often convenient to express the results in terms of loop current I_a and/or field intensity E_a from the reference tower if all the power were radiated from that tower with no losses. Thus,

$$(8)\quad \begin{cases} I_a = \sqrt{P_a/R_{oo}} \\ E_a = 37.25\ \sqrt{P/R_{oo}}\ (1 - \cos G_o) \end{cases}$$

Substituting (8) into (5a) and (1b), the pattern constants and RMS are obtained in terms of E_a and I_a.

$$
(9)\quad \begin{cases} I_o = I_a/r_L \\ E_o = E_a/r_L \\ E_{RMS} = \left(e/r_L\right) E_a = \left[e/\sqrt{r^2+a^2R_L}\right] E_a \end{cases}
$$

The above form in conjunction with Fig. 1 is very convenient for calculating the pattern constants and E_{RMS}.

To assist in the calculation of the constants e, r, and a, several curves have been prepared. Fig. 1 contains curves of the self resistance R_{nn} and E_a versus height G. Unfortunately, there are not available many curves of mutual resistance between unequal height antennas. This mutual resistance may be calculated from either a lengthy formula[1] or graphically[2].

Horizontal Field Gain and Power Efficiency

The form of equation (9) is very useful for studying the effect of the ground and tower loss R_L upon the E_{RMS}, since e, r, and a are independent of R_L. With this form the calculations become simple.

From the basic equations in the preceding section other formulas of interest may be derived. One such formula, for example, is that for the horizontal gain of the array, which is defined by,

$$
(10)\qquad g = E_{RMS}/E_a = e/r_L = e/\sqrt{r^2+a^2R_L}
$$

In other words, the array would radiate g^2 times as much useful power in the horizontal plane as would a single tower of reference height and no losses, if all the power were concentrated in that one tower.

Another useful auxiliary is that for the power efficiency of the array:

$$
(11)\qquad \begin{aligned} \eta &= P_R/(P_R+P_L) = r^2/r_L{}^2 = r^2/(r^2+a^2R_L) \\ &= 1/\left[1+R_L(a/r)^2\right] \end{aligned}
$$

1 "Mutual Impedance Between Unequal Height Antennas", by Cox, IRE, Nov., 1947.

2 "Directional Antennas", by C. E. Smith, P. 84.

In equation (11) it is observed that the effect of ground and tower loss is contained only in R_L, while the independent factor a/r represents the contribution of the inherent design of the array. This factor a/r determines the total effect of the array design upon the power efficiency. Thus, with equation (11) the effects of antenna design and/or tower plus ground loss may be studied quite easily.

Most arrays are designed to have equal height elements and for this special case the preceding formulas can be simplified on account of the following equalities:

$$(12)\quad \left\{ \begin{aligned} F_n &= M_n \\ G_n &= G_o \\ R_{nn} &= R_{oo} \end{aligned} \right.$$

The working constants become,

$$\left\{ \begin{aligned} e^2 &= \sum_{n=1}^{N} \sum_{p=1}^{N} M_n M_p\, J_o(S_{np}) \cos(\psi_n - \psi_p) \\ r^2 &= \sum_{n=1}^{N} \sum_{p=1}^{N} M_n M_p\, R_{np}/R_{oo} \cos(\psi_n - \psi_p) \\ a^2 &= \frac{\sin^2 G_o}{R_{oo}} \sum_{n=1}^{N} M_n{}^2 = \frac{\sin^2 H_o}{R_{oo}}\, s^2 \\ s^2 &= \sum_{n=1}^{N} M_n{}^2 \end{aligned} \right.$$

where s is defined above. For any specific case these constants may be developed and calculated simultaneously, since the corresponding terms of a^2 and r^2 contain the same factors except one.

The advantage of using the ratio R_{np}/R_{oo} lies in the fact that this ratio is almost independent of antenna height, making it easier to interpolate for intermediate heights as well as making the factor r substantially independent of height. R_{np}/R_{oo} for equal height antennas as well as $J_o(S_{np})$ are plotted in Fig. 2. These curves of R_{np}/R_{oo} have been calculated by Commission personnel.

To digress for a moment, a useful approximation to the mutual resistance between two towers of unequal heights G_1 and G_2 is:

$$R_m(G_1, G_2) \cong \sqrt{\delta_{11}\,\delta_{22}\,R_{11}\,R_{22}} = \sqrt{R_m(G_1, G_1)\,R_m(G_2, G_2)} \tag{14}$$

Where R_{nn} and δ_{nn} are the self resistance and the ratio of mutual to self resistances, respectively, for two towers of equal height G_n, and $R_m(G_n, G_p)$ is the mutual resistance between two towers of unequal heights G_n and G_p. The derivation of this approximation is as follows: Define, in analogy with lumped circuits, the ratio of the mutual resistance between two antennas to the geometric mean of their self resistances as the coefficient of coupling between the antennas. Then, it is plausible and indeed approximately true that the coefficient of coupling between two antennas of unequal height is the geometric mean between the coefficients obtained by assuming them to be both first of the one height and then both of the second height; i.e.,

$$\delta_{12} \cong \sqrt{\delta_{11}\,\delta_{22}} \tag{14a}$$

Using the curves of Figures 1 and 2 for the values of R_{11}, R_{22}, δ_{11}, and δ_{22} the mutual resistance between two unequal height antennas may thus be approximated rather quickly. The resultant approximation is accurate within several ohms but the reader is cautioned not to use it where accuracy is required.

The design efficiency factor, a/r, of (11) is rather interesting for this case of equal height elements and reduces to,

$$a/r = (\sin G_o/\sqrt{R_{oo}})(s/r) \tag{15}$$

The effect of antenna height is practically all contained in the term $\sin G_o/\sqrt{R_{oo}}$ since s is independent and r almost independent of antenna height. The factor s/r represents the contribution of the spacing between the antenna elements upon the system efficiency.

The term s/r may be further subdivided into,

$$s/r = (s/e)\ (e/r) \tag{16}$$

From (9) it is seen that e/r is the horizontal field intensity gain with no losses and almost never exceeds a value of 1.5. On the other hand, s/e is the ratio of RSS (root sum square) to RMS (root mean square) fields and may get quite large so that this index s/e is a fairly good index of the design with respect to power efficiency, a large ratio of s/e indicating a poor efficiency. Mr. Glenn D. Gillett has proposed the use of this ratio s/e as an index of efficiency of an array. This ratio is relatively easy to calculate and is therefore very convenient. However, it should be realized that it is not a completely accurate index since it neglects the effects of antenna height upon power efficiency. Thus, for two arrays of different heights the same value of s/e would not denote the same efficiency for a given loss but might indicate whether or not the systems tended to be inefficient. As previously stated the ratio a/r is the true index of the design efficiency.

Tolerance and Stability

It is frequently necessary to check an array to ascertain whether or not it will exceed given tolerance values in certain specified directions as the various currents in the towers vary with amplitude and/or phase. The best approach to this problem is to add graphically the individual vector fields in the critical directions. On this type of plot it is comparatively easy to study and visualize the relative effect upon the resultant vector of varying the amplitudes and/or phases of the currents in the individual towers. The resultant vector field is:

$$\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2 + \ldots\ldots + \mathcal{E}_N$$

(17)
$$= \sum_{n=1}^{N} \mathcal{E}_n$$

where $\mathcal{E}_n$ represents the vector field from tower n and is given by,

(18) $$\mathcal{E}_n = E_n f_n(\theta) e^{j\left[\psi_n + S_n \cos\theta \cos(\phi - \phi_n)\right]}$$

Where,

(19) $$f_n(\theta) = \frac{\cos(G_n \sin\theta) - \cos G_n}{\cos\theta (1 - \cos G_n)}$$

and

$f_n(\theta)$ ≡ vertical antenna form factor of tower n (See Fig. 4).

θ ≡ Angle of elevation from horizontal plane.

S_n ≡ spacing in electrical degrees of tower n from reference point.

ϕ_n ≡ horizontal orientation angle of tower n from the reference line.

ϕ ≡ horizontal angle from reference line.
$f(\theta)$ is plotted in Fig. 3.

The geometry is shown in Fig. 4.

When it is required to plot the pattern of an array, the magnitude of the field at any point is found by taking a dot product of $\mathcal{E}$. Thus:

$$E^2 = \mathcal{E}\cdot\mathcal{E} = \sum_{n=1}^{N} \sum_{p=1}^{N} \mathcal{E}_n \cdot \mathcal{E}_p$$

$$= \sum_{n=1}^{N} \sum_{p=1}^{N} E_n E_p f_n(\theta) f_p(\theta) \cos\left[(\psi_n - \psi_p) + S_{np} \cos\theta\cos(\phi-\phi_{np})\right]$$

$$= E_o^2 \sum_{n=1}^{N} \sum_{p=1}^{N} F_n F_p f_n(\theta) f_p(\theta) \cos\left[(\psi_n-\psi_p) + S_{np} \cos\theta\cos(\phi-\phi_{np})\right]$$

(20)

$$= E_o^2 \Big\{ F_1^2 f_1^2(\theta) + F_2^2 f_2^2(\theta) + \dots + F_N^2 f_N^2(\theta)$$

$$+ 2F_1F_2f_1(\theta)f_2(\theta) \cos\left[(\psi_1 - \psi_2) + S_{12}\cos\theta\cos(\phi - \phi_{12})\right]$$

$$+ \dots + 2F_1F_Nf_1(\theta)f_N(\theta) \cos\left[(\psi_1 - \psi_N) + S_{1N} \cos\theta\cos(\phi - \phi_{1N})\right]$$

$$+ 2F_2F_3f_2(\theta)f_3(\theta) \cos\left[(\psi_2 - \psi_3) + S_{23} \cos\theta\cos(\phi - \phi_{23})\right]$$

$$+ \dots + 2F_2F_Nf_2(\theta)f_N(\theta) \cos\left[(\psi_2 - \psi_N) + S_{2N} \cos\theta\cos(\phi - \phi_{2N})\right]$$

$$+ \dots 2F_{N-1} F_N f_{N-1}(\theta)f_N(\theta) \cos\left[(\psi_{N-1} - \psi_N) + S_{(N-1)N} \cos\theta\cos(\phi-\phi_{(N-1)N})\right]\Big\}$$

For the special case of equal height antennas,

(18a) $$\mathcal{E} = E_o f(\theta) \sum_{n=1}^{N} M_n \mathcal{E}^{j\left[\psi_n + S_n \cos\theta\cos(\phi - \phi_n)\right]}$$

(20a) $$E = E_o f(\theta)\left\{ \sum_{n=1}^{N} \sum_{p=1}^{N} M_n M_p \cos\left[(\psi_n - \psi_p) + S_{np} \cos\theta\cos(\phi - \phi_{np})\right]\right\}^{\frac{1}{2}}$$

Where $S_{np}\mathcal{E}^{j\phi_{np}}$ is the vector spacing from towers <u>p to n</u>. It should be noted that in using (20) or (20a),

$$S_{np} = S_{pn}$$

(21)

$$\emptyset_{np} = \emptyset_{pn} + \pi$$

Usually the terms of (20) or (20a) can be combined into simpler trigonometrical form.

Illustrative Problem

A specific design will now be checked in order to demonstrate the use of the formulas and concepts previously developed. The equal height array shown below will be checked.

#1 60° #2 60° #3

135° true

$0.5 \angle 11.4^\circ$ $0.753 \angle 176.6^\circ$ $0.5 \angle -11.4^\circ$

The given constants are,

$G_{oo} = 65^\circ$ $M_1 = F_1 = 0.5$ $M_3 = F_3 = 0.5$

$S_{12} = S_{23} = 60^\circ$ $\psi_1 = 11.4^\circ$ $\psi_3 = -11.4^\circ$

$S_{13} = 120^\circ$ $M_2 = F_2 = 0.753$ $P = 1$ kw.

$\psi_2 = 176.6^\circ$

In the direction of 165° true a maximum horizontal tolerance field of 60 mv/m is permitted. The working constants are developed in parallel from equation (13).

$$\left\{\begin{aligned} e^2 &= M_1^2 + M_2^2 + M_3^2 + 2M_1M_2\, J_o(S_{12}) \cos(\psi_1 - \psi_2) \\ &\quad + 2M_1M_3\, J_o(S_{13}) \cos(\psi_1 - \psi_3) + 2M_2M_3\, J_o(S_{23}) \cos(\psi_2 - \psi_3) \\ e^2 &= (0.5)^2 + (0.753)^2 + (0.5)^2 + 2(0.5)(0.753)(.740) \left[\cos(-165.2^\circ)\right] \end{aligned}\right.$$

$$+ 2(0.5)(0.5)(.175) \cos 22.8^\circ + 2(0.753)(0.5)(.740) \cos 188.0^\circ = .057$$

$$\left\{\begin{aligned} r^2 &= M_1{}^2 + M_2{}^2 + M_3{}^2 + 2M_1M_2\, R_{12}/R_{oo} \cos(\psi_1 - \psi_2) \\ &\quad + 2M_1M_3\, R_{13}/R_{oo} \cos(\psi_1 - \psi_3) + 2M_2M_3\, R_{23}/R_{oo} \cos(\psi_2 - \psi_3) \\ r^2 &= (0.5)^2 + (0.753)^2 + (0.5)^2 + 2(0.5)(0.753)(.800) \cos(-165.2^\circ) \\ &\quad + 2(0.5)(0.5)(.295) \cos 22.8^\circ + 2(0.753)(0.5)(.800) \cos 165.2^\circ = .024 \end{aligned}\right.$$

$$\left\{\begin{aligned} s^2 &= M_1{}^2 + M_2{}^2 + M_3{}^2 \\ s^2 &= \left[(0.5)^2 + (0.753)^2 + (0.5)^2\right] = 1.067 \end{aligned}\right.$$

$$\left\{\begin{aligned} a^2 &= \frac{\sin^2 G_0}{R_{oo}}\; s^2 \\ a^2 &= \frac{(.906)^2}{9.7}\;(1.067) = .0687 \end{aligned}\right.$$

From the above,

$e = 0.239$ $\qquad$ $s = 1.033$

$r = 0.155$ $\qquad$ $a = 0.262$

The "Gillett" ratio is,

$s/e = 4.33$

and the design efficiency factor is,

$a/r = 1.69$

Both ratios are high indicating an inefficient system with consequent instability. From Fig. 1, E_a = 190 mv/m and using equations 7, 9, and 10, we get the following table:

R_L	a^2R_L	$r_L{}^2$	r_L	E_{rms}	Eff.
0.5	.034	.058	.241	188	.414
1	.069	.093	.305	149	.258
2	.137	.161	.402	113	.149

It is noted that a <u>very</u> good ground system would be necessary to meet the minimum required E_{RMS} of 175 mv/m. Also, a slight change in R_L would cause a large change in the E_{RMS} and the efficiency of the system, causing instability.

Assuming that an R_L of 0.5 could be maintained, the pattern constant (equation 9)

$$E_o = E_a/r_L = 190/.241 = 788$$

The horizontal field gain is,

$$g = e/r_L = 0.992$$

The resultant vector field is,

$$\mathcal{E} = E_o\, f(\theta) \Big\{ F_1 \; \varepsilon^{j\left[\psi_1 + S_1 \cos\theta \cos(\phi - \phi_1)\right]} + F_2\, \varepsilon^{j\left[\psi_2 + S_2 \cos\theta \cos(\phi - \phi_2)\right]} + F_3\, \varepsilon^{j\left[\psi_3 + S_3 \cos\theta \cos(\phi - \phi_3)\right]} \Big\}$$

$$\mathcal{E} = 788\, f(\theta) \Big\{ 0.5 \; \varepsilon^{j\left[11.4 + 60 \cos\theta \cos(\phi - \pi)\right]} + 0.753\, \varepsilon^{j\,176.6} + 0.5\, \varepsilon^{j\left[-11.4 + 60 \cos\theta \cos(\phi)\right]} \Big\}$$

$$= 788\, f(\theta) \Big\{ 0.753\, \varepsilon^{j\,176.6} + \cos\left[11.4 - 60 \cos\theta \cos\phi\right] \Big\}$$

where the center tower is the reference point and the reference line is the line of towers at 135° true. The magnitude of the field in any direction may be obtained from the above expression of $\mathcal{E}$ but for illustrative purposes is derived from equation 20.

$$E = E_o f(\theta) \Big\{ M_1^2 + M_2^2 + M_3^2 + 2M_1M_2 \cos\left[(\psi_1 - \psi_2) + S_{12} \cos\theta \cos(\phi - \phi_{12})\right] + 2M_1M_3 \cos\left[(\psi_1 - \psi_3) + S_{13} \cos\theta \cos(\phi - \phi_{13})\right] + 2M_2M_3 \cos\left[(\psi_2 - \psi_3) + S_{23} \cos\theta \cos(\phi - \phi_{23})\right] \Big\}^{\frac{1}{2}}$$

$$E = 788\, f(\theta) \Big\{ \overline{0.5}^2 + \overline{0.753}^2 + \overline{0.5}^2 + 2(0.5)^{(0.753)} \cos\left[(11.4 - 176.6) + 60 \cos\theta \cos\phi\right] + (2)\overline{0.5}^2 \cos\left[22.4 - 120 \cos\theta \cos\phi\right]$$

$$+ 2(0.753)(0.5) \cos \left[188.0 - 60 \cos \theta \cos \phi\right]\Big\}^{\frac{1}{2}}$$

$$= 788\, f(\theta) \Big\{1.067 + 0.753 \cos \left[165.2 + 60 \cos \theta \cos \phi\right]$$

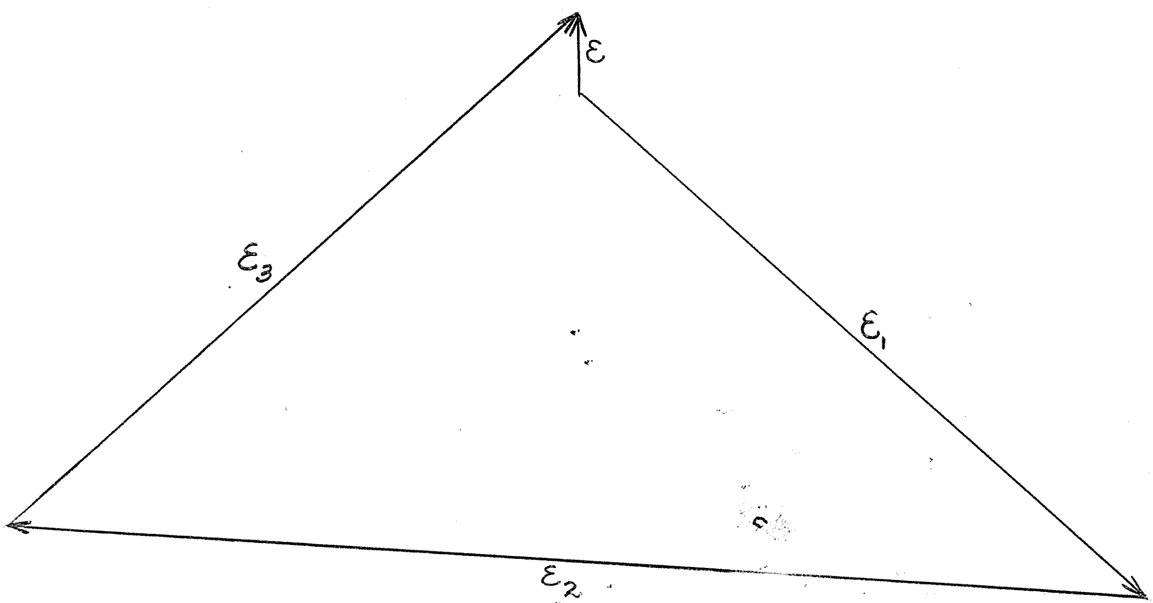
$$+ 0.5 \cos \left[22.4 - 120 \cos \theta \cos \phi\right]$$

$$+ 0.753 \cos \left[188 - 60 \cos \theta \cos \phi\right]\Big\}^{\frac{1}{2}}$$

In the direction of 165° true a maximum E of 60 mv/m is allowed. For $\phi = 30^\circ$ and $\theta = 0$,

$$\mathcal{E}_1 = 394\, \varepsilon^{-j\ 40.6} \qquad \mathcal{E}_2 = 594\, \varepsilon^{j\ 176.6} \qquad \mathcal{E}_3 = 394\, \varepsilon^{j\ 40.6}$$

The vector diagram is shown below. The effect of changing the amplitude and/or phase of any vector can be studied quite easily.



From this vector diagram it is noted that the magnitude E of the vector $\mathcal{E}$ is particularly sensitive to changes in phase and magnitude of $\mathcal{E}_2$, since $\mathcal{E}_1$ and $\mathcal{E}_3$ are smaller and complex conjugates. A combination of 5 percent increase in I_2 and a 2 degree decrease in ψ_2 would change E from 37.1 to 64.5 mv/m, exceeding the MEOV of 60 mv/m in this direction. The worst possible condition, of course, would occur with the combination for which every magnitude and phase changes simultaneously in the direction of increasing E.

Conclusion.

Several useful concepts and constants have been developed for the purpose of reducing a check on the design of a directional array to an organized procedure. No attempt has been made to develop the design theory itself. It is unfortunate that the mutual resistance curves are not very complete and some day it is hoped that a more complete set will be available.

Acknowledgment

The constructive criticism and helpful suggestions of E. F. Vandivere, Jr. and H. E. Slone are gratefully acknowledged. George L. Gadea calculated the $\frac{R_m}{R_{oo}}$ curves of Fig. 1 and Table I.

Harry Fine

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Approved: Edward W. Allen, Jr.

Edward W. Allen, Jr.
Chief, Technical Research Division

TABLE IA

S	R_m/R_{oo} h = 45°	h = 90°	h = 135°	h = 180°	h = 225°	h = 270°	S
45°	0.87999	0.87894	0.87331	0.86759	0.86459	0.91178	45°
70	0.72301	0.71781	0.70769	0.69052	0.68886		70
90	0.56541	0.55788	0.54241	0.51663	0.51477	0.68436	90
115	0.34708	0.33739	0.31579	0.27961	0.27884		115
135	0.17345	0.16083	0.13526	0.09265	0.09415	0.37868	135
160	-0.02509	-0.03956	-0.06835	-0.12079	-0.10874		160
180	-0.15662	-0.17137	-0.19986	-0.24534	-0.23412	0.08111	180
200	-0.25297	-0.26867	-0.29488	-0.33560	-0.31810		200
225	-0.32454	-0.33580	-0.35623	-0.38561	-0.35976	-0.13545	225
233				-0.38690			233
245		-0.34537					245
250	-0.33525		-0.35401	-0.36743			250
255					-0.32145		255
270	-0.30494	-0.30768	-0.31128	-0.31058	-0.27222	-0.23614	270
290	-0.24725	-0.24569	-0.24088	-0.22541	-0.18460	-0.24557	290
315	-0.14930	-0.14291	-0.12811	-0.09618	-0.05622	-0.22834	315
340	-0.04063	-0.03007	-0.00735	0.03740	0.07087		340
360	0.03987	0.05487	0.08155	0.13107	0.15683	-0.15152	360
385	0.12680	0.14071	0.16681	0.21892	0.22965		385
405	0.17274	0.18602	0.21284	0.25786	0.25584	-0.05546	405
420				0.26701			420
425	0.19525	0.20699	0.23003	0.26610	0.24995		425
430		0.20847		0.26422		-0.01103	430
450	0.19060	0.19903	0.21450	0.23537	0.20215	0.01712	450
470		0.16684		0.17883		0.03716	470
495	0.10720	0.10746	0.10598	0.09463	0.04089	0.05078	495
520						0.05345	520
540		-0.02581		-0.07750		0.04956	540

TABLE IB

S	$J_o(s)$
20°	0.96977
40	0.88181
60	0.74407
80	0.56889
100	0.37174
120	0.16981
140	-0.02036
160	-0.18174
180	-0.30435
200	-0.37887
220	-0.40274
240	-0.37806
260	-0.31157
280	-0.21382
300	-0.09791
320	0.02027
340	0.13197
360	0.22024
380	0.27789
400	0.29993
420	0.28562
440	0.23832
460	0.16503
480	0.07510
500	-0.01979
520	-0.10862
540	-0.18124

TABLE IC

H_1	R_{oo}
45°	3.3597
90	36.5623
135	92.89703
180	99.5372
225	53.26457
270	52.7431

TABLES IB and IC

TABLE IB

S	$J_0(s)$
20°	0.96977
40	0.88181
60	0.74407
80	0.56889
100	0.37174
120	0.16981
140	−0.02036
160	−0.18174
180	−0.30435
200	−0.37887
220	−0.40274
240	−0.37806
260	−0.31157
280	−0.21382
300	−0.09791
320	0.02027
340	0.13197
360	0.22024
380	0.27789
400	0.29993
420	0.28562
440	0.23832
460	0.16503
480	0.07510
500	−0.01979
520	−0.10862
540	−0.18124

TABLE IC

H	R_{oo}
45°	3.3597
90	36.5623
135	92.89703
180	99.5372
225	53.26457
270	52.7431

TABLE 1-D

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VERTICAL RADIATION CHARACTERISTICS

$$f(\theta) = \frac{\cos (G \sin \theta) - \cos G}{(1 - \cos G) \cos \theta}$$

θ = Vertical Angle

G = Antenna height - electrical degrees

G°	50	55	60	65	70	75	80	85	90	95
θ					f(θ)					
0°	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	.9829	.9824	.9819	.9815	.9809	.9803	.9795	.9788	.9780	.9769
20	.9324	.9309	.9289	.9272	.9251	.9226	.9200	.9172	.9143	.9110
30	.8519	.8486	.8453	.8416	.8375	.8328	.8277	.8224	.8165	.8102
40	.7449	.7410	.7358	.7305	.7244	.7179	.7109	.7103	.6946	.6855
50	.6185	.6129	.6073	.6008	.5936	.5861	.5777	.5686	.5591	.5486
60	.4759	.4705	.4648	.4586	.4517	.4441	.4361	.4270	.4178	.4078
70	.3216	.3189	.3140	.3089	.3031	.2970	.2905	.2841	.2766	.2687
80	.1629	.1622	.1578	.1557	.1515	.1492	.1456	.1407	.1377	.1330
90	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

G°	100	105	110	115	120	125	130	135	140	145
θ					f(θ)					
0°	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	.9759	.9749	.9737	.9725	.9712	.9697	.9681	.9663	.9645	.9624
20	.9074	.9035	.8993	.8947	.8898	.8845	.8788	.8725	.8657	.8584
30	.8033	.7959	.7878	.7791	.7698	.7597	.7489	.7372	.7245	.7109
40	.6759	.6655	.6540	.6420	.6288	.6157	.5999	.5837	.5664	.5477
50	.5372	.5253	.5124	.4987	.4838	.4680	.4511	.4330	.4137	.3931
60	.3969	.3854	.3730	.3599	.3460	.3310	.3151	.2981	.2802	.2611
70	.2598	.2509	.2412	.2310	.2205	.2090	.1969	.1838	.1699	.1554
80	.1281	.1236	.1180	.1130	.1064	.1007	.0940	.0867	.0796	.0719
90	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

G°	150	160	170	180	190	200	210	220	230	240
θ					f(θ)					
0°	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	.9602	.9551	.9491	.9418	.9330	.9222	.9090	.8924	.8580	.8442
20	.8504	.8324	.8110	.7855	.7548	.7175	.6718	.6151	.5438	.4525
30	.6960	.6628	.6237	.5774	.5222	.4561	.3757	.2772	.1548	.0000
40	.5276	.4828	.4305	.3696	.2979	.2129	.1112	.0116	.1620	.3489
50	.3710	.3220	.2657	.2008	.1256	.0378	.0656	.1885	.3362	.5163
60	.2407	.1960	.1451	.0873	.0212	.0550	.1431	.2460	.3673	.5124
70	.1399	.1064	.0687	.0260	.0220	.0764	.1388	.2106	.2935	.3908
80	.0633	.0457	.0255	.0032	.0218	.0499	.0818	.1181	.1599	.2078
90	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

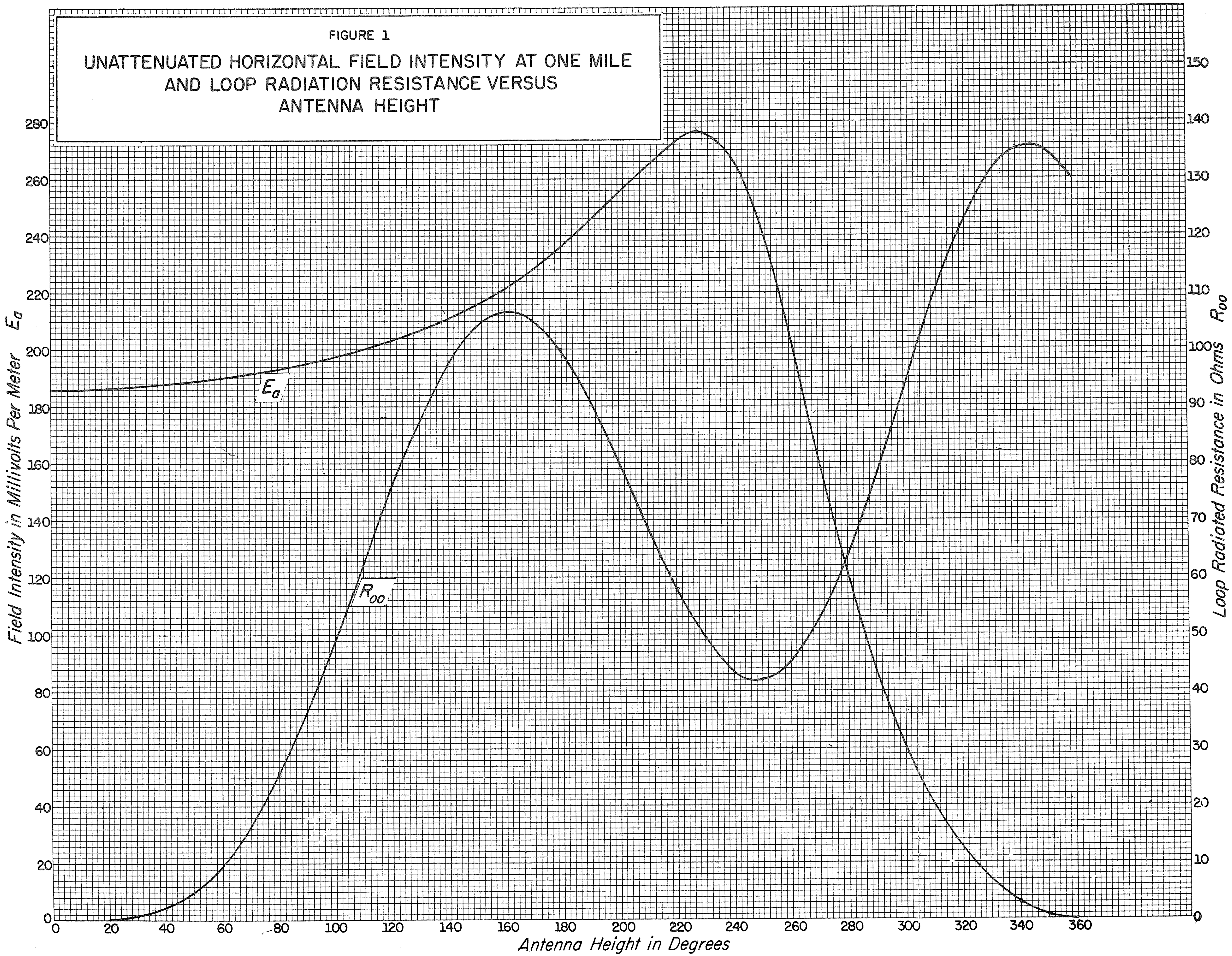


FIGURE 1

UNATTENUATED HORIZONTAL FIELD INTENSITY AT ONE MILE AND LOOP RADIATION RESISTANCE VERSUS ANTENNA HEIGHT

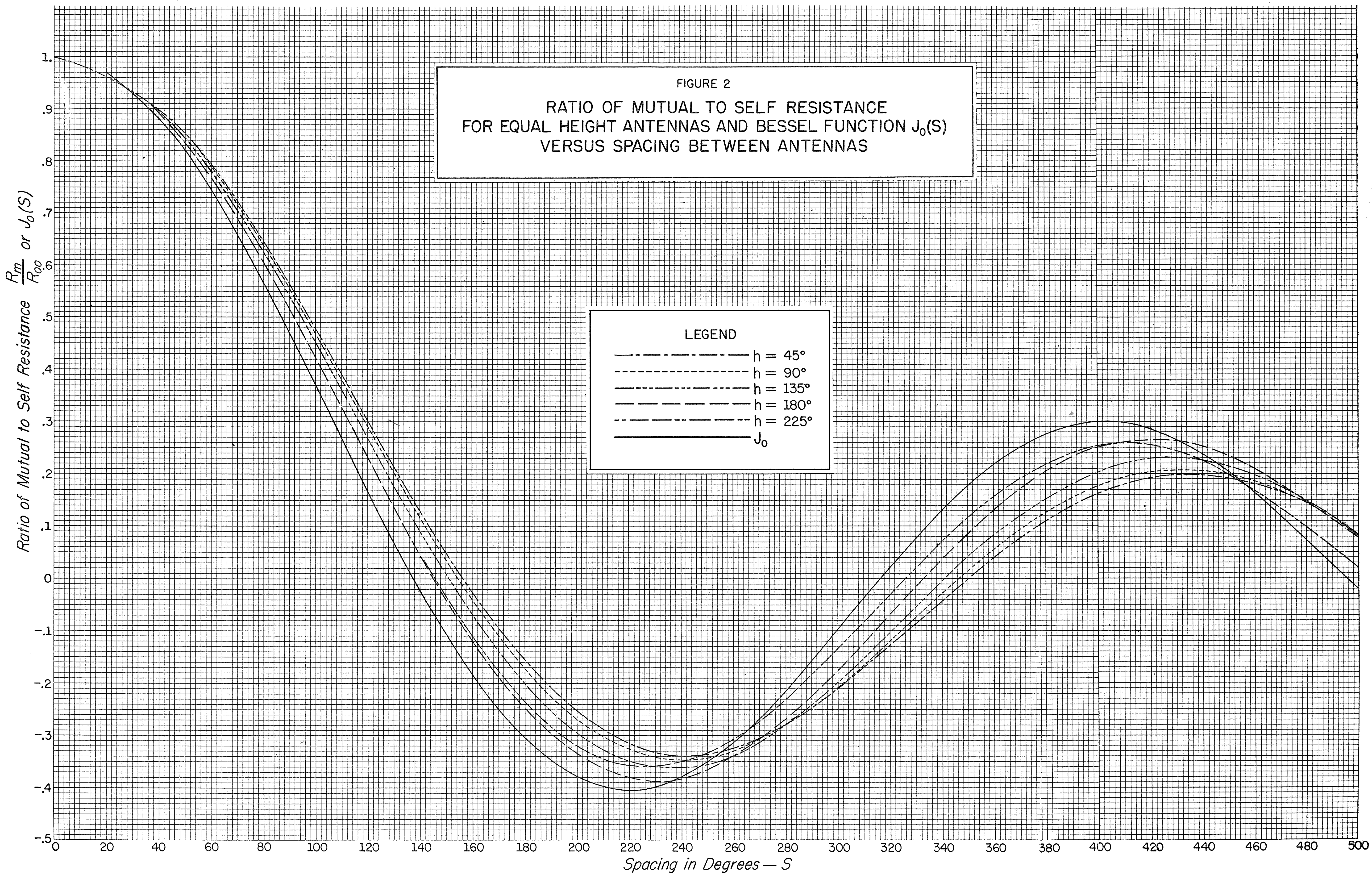
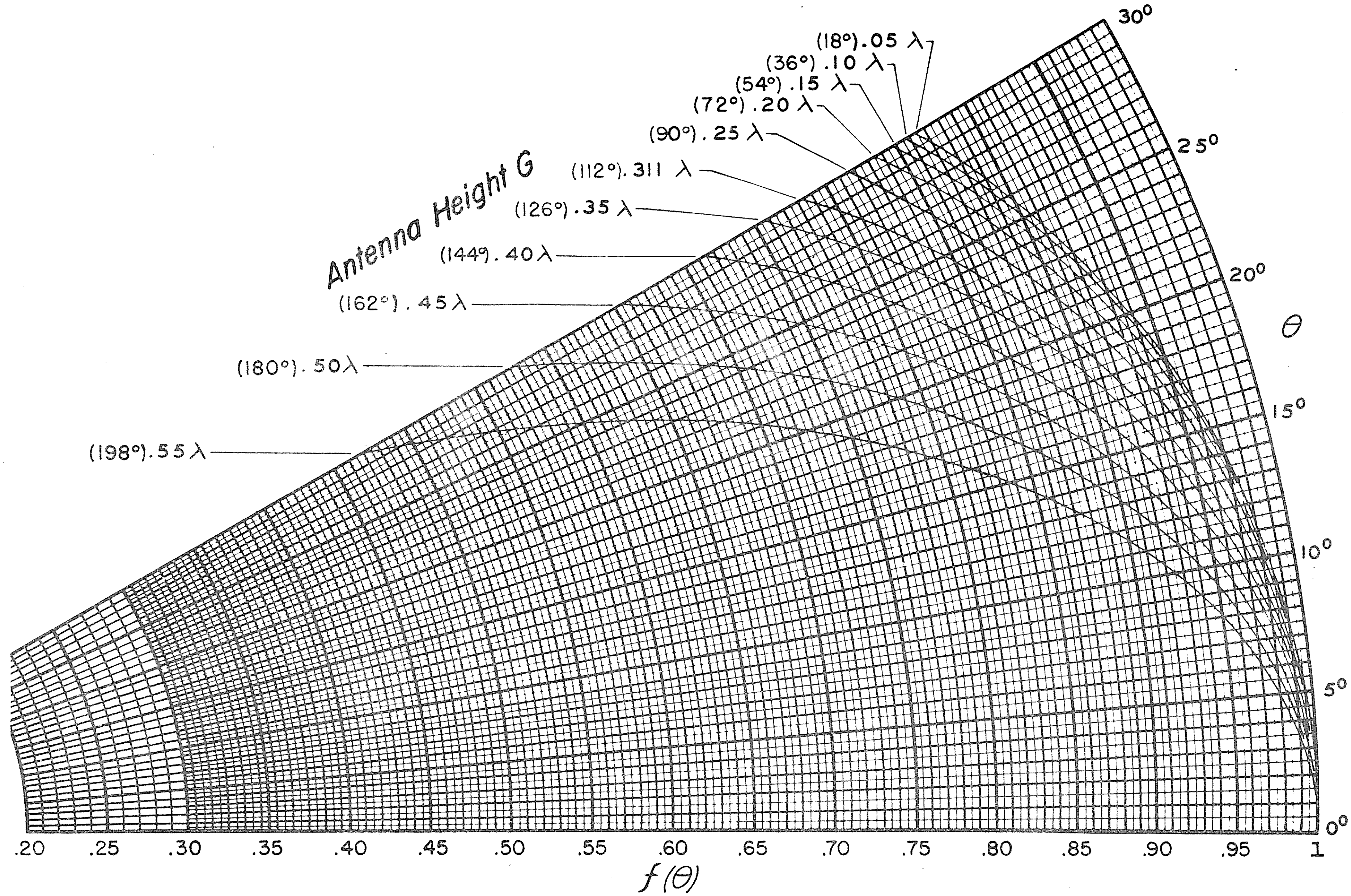


FIGURE 2
RATIO OF MUTUAL TO SELF RESISTANCE FOR EQUAL HEIGHT ANTENNAS AND BESSEL FUNCTION $J_0(S)$ VERSUS SPACING BETWEEN ANTENNAS
LEGEND
h = 45°
h = 90°
h = 135°
h = 180°
h = 225°
J_0
Ratio of Mutual to Self Resistance $\frac{R_m}{R_{00}}$ or $J_0(S)$
Spacing in Degrees — S
1.
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
-.1
-.2
-.3
-.4
-.5
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
320
340
360
380
400
420
440
460
480
500



ANTENNA FORM FACTOR VERSUS ANGLE OF ELEVATION

FIGURE 3

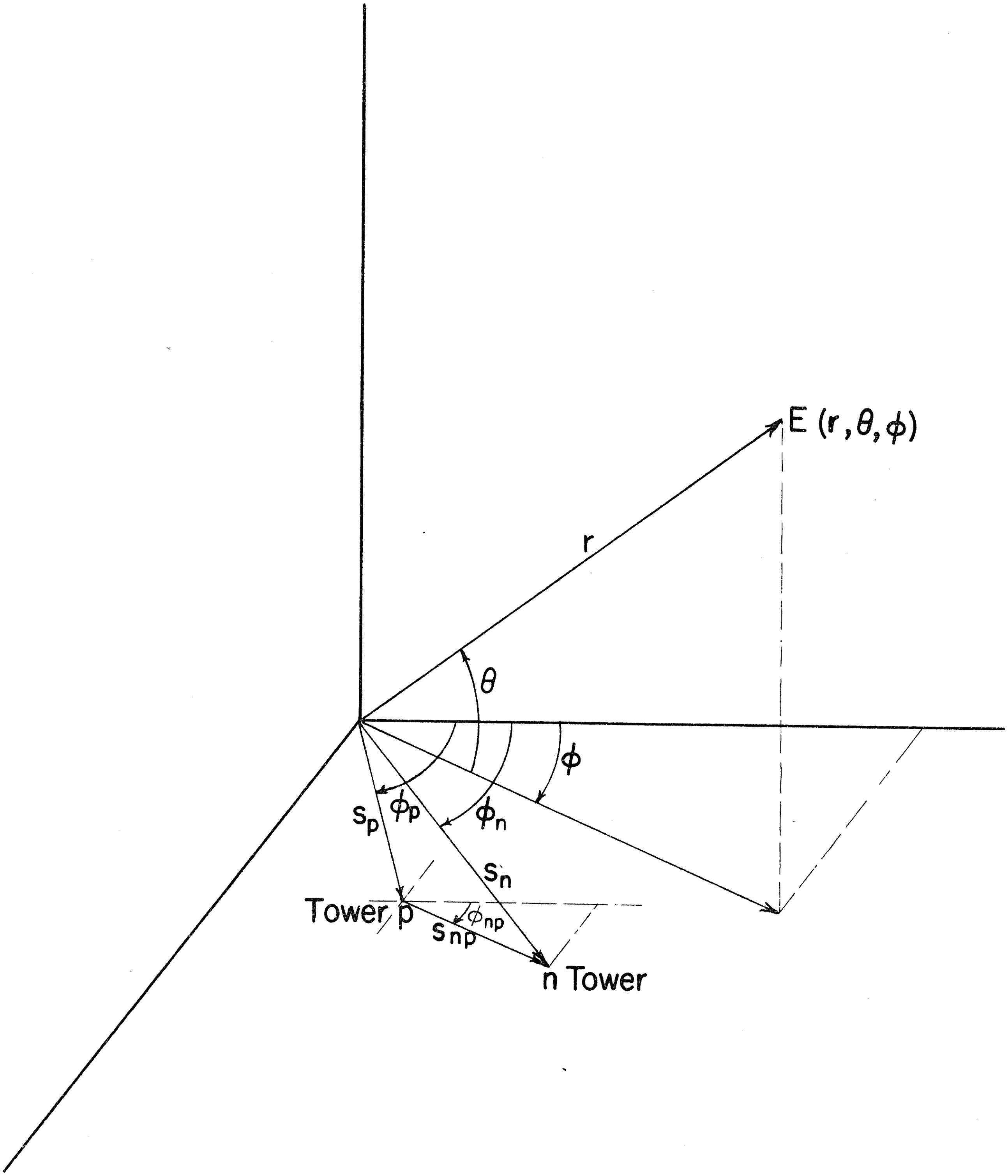


FIGURE 4